

**The Association  
of  
Engineering and Shipbuilding  
Draughtsmen.**

---

**PHOTOELASTICITY AND THE  
ENGINEER.**

---

**By K. J. RAWSON, Wh.Sch., R.C.N.C.**

---

**Published by The Association of Engineering and Shipbuilding Draughtsmen,  
Onslow Hall, Little Green, Richmond, Surrey.**

---

**SESSION 1953-54.**

**Price 3/-**

**Printed by Milne, Tannahill & Methven, Ltd. (T.U.), 12-14 Mill Street, Perth.**

## ADVICE TO INTENDING AUTHORS OF A.E.S.D. PRINTED PAMPHLETS.

---

Pamphlets submitted to the National Technical Sub-Committee for consideration with a view to publication in this series should not exceed 10,000 to 15,000 words and about 20 illustrations. The aim should be the presentation of the subject clearly and concisely, avoiding digressions and redundancy. Manuscripts are to be written in the third person. Copies of an article entitled "Hints on the Writing of Technical Articles" can be obtained from the Editor of *The Draughtsman*.

Drawings for illustrations should be done either on a good plain white paper or tracing cloth, deep black Indian ink being used. For ordinary purposes they should be made about one-and-a-half times the intended finished size, and it should be arranged that wherever possible these shall not be greater than a single full page of the pamphlet, as folded pages are objectionable, although, upon occasion, unavoidable. Where drawings are made larger, involving a greater reduction, the lines should be made slightly heavier and the printing rather larger than normal, as the greater reduction tends to make the lines appear faint and the printing excessively small in the reproduction. In the case of charts or curves set out on squared paper, either all the squares should be inked in, or the chart or curve should be retraced and the requisite squares inked in. Figures should be as self-evident as possible. Data should be presented in graphical form. Extensive tabular matter, if unavoidable, should be made into appendices.

Authors of pamphlets are requested to adhere to the standard symbols of the British Standards Institution, where lists of such standard symbols have been issued, as in the case of the electrical and other industries, and also to the *British Standard Engineering Symbols and Abbreviations*, No. 560, published by the B.S.I. in 1934 at 5/-. Attention might also be given to mathematical notation, where alternative methods exist, to ensure the minimum trouble in setting up by the printer.

The value of the pamphlet will be enhanced by stating where further information on the subject can be obtained. This should be given in the form of footnotes or a bibliography, including the name and initials of the author, title, publisher, and year of publication. When periodicals are referred to, volume and page also should be given. References should be checked carefully.

Manuscripts, in the first instance, should be submitted to the Editor, *The Draughtsman*, Onslow Hall, Little Green, Richmond, Surrey.

For Pamphlets, a grant of £20 will be made to the Author, but special consideration will be given in the case of much larger pamphlets which may involve more than the usual amount of preparation.

*The Publishers accept no responsibility for the formulae or opinions  
expressed in their Technical Publications.*

The Association  
of  
Engineering and Shipbuilding  
Draughtsmen.

---

**PHOTOELASTICITY AND THE  
ENGINEER.**

---

By K. J. RAWSON, Wh.Sch., R.C.N.C.

---

Published by The Association of Engineering and Shipbuilding Draughtsmen,  
Onslow Hall, Little Green, Richmond, Surrey.

---

SESSION 1953-54.

# CONTENTS.

---

- I. INTRODUCTION.
- II. THE NATURE OF POLARISED LIGHT.
  - Reflection and Refraction.
  - Double Refraction.
  - Interference.
  - Circularly Polarised Light.
- III. PHOTOELASTICITY.
  - The Laws of Photoelasticity.
  - Observations of Interference.
  - The Isochromatic Fringes.
  - The Isoclinic Lines.
  - Isotropic Points.
  - Stress Trajectories.
  - Application of Results.
  - Limitations of Elementary Practice.
- IV. EXPERIMENTAL TECHNIQUES.
  - The Plane Polariscopes.
  - The Circular Polariscopes.
  - The Use of Compensators.
  - The Portable Polariscopes.
  - Three Dimensional Photoelasticity.
  - The Scattered Light Technique.
- V. DETERMINATION OF THE SEPARATE STRESSES.
  - Lateral Extensometers.
  - Integration Methods.
  - Relaxation Method.
- VI. MATERIALS.
  - Material Fringe Value.
  - Creep.
  - Edge Effects.
  - Bench Materials.
  - Model Materials.
  - Table of Constants.
- VII. BIBLIOGRAPHY.

# PHOTOELASTICITY AND THE ENGINEER.

By K. J. RAWSON, Wh.Sch., R.C.N.C.

---

## I. INTRODUCTION.

THE object of any form of stress analysis is to produce a structure which is sufficiently strong for its purpose, and yet as light as possible. To this end there are, in general, three approaches :—

- (i) To analyse the structure mathematically, using the classical theories and assumptions of elasticity or plasticity. With finite boundaries of even simple engineering shapes, this soon becomes exceedingly complicated and the simplified assumptions necessary for solution no longer hold.
- (ii) The full size structure may be built and a stress analysis carried out by means of strain gauges with the structure under load, improving it as a result of measurements taken. This method can be expensive and laborious but is occasionally the only approach possible.
- (iii) A model of the structure may be built in an exact or simplified version, in a material in which the stresses may be detected. If a steel model can be made (and if the scale be small, this is frequently not practicable), the highest stress may occur at a point or over such a short length that the use of a strain gauge is not possible. Often, such a model can be readily made in a plastic transparent material, in which the stresses can be found optically. It is this science of determining stresses in a transparent material by observing the effects of the passage of polarised light through it, that is known as photoelasticity.

Such effects were first observed in glass by Sir David Brewster at the beginning of the 19th Century and were of interest only to the physicist for nearly a hundred years. At the turn of this century Mesnager, a Frenchman, began the development of the technique, but it was not until the two great pioneers, Coker and Filon, published their "Treatise on Photoelasticity" in 1930, that the subject became of concerning interest to the engineer. Since that time, a number of books have been published, some of which are listed in the bibliography at the end of this pamphlet, and great advances have been made in the theory and practice of photoelasticity.

To-day, the science is widely used in all branches of engineering and has been responsible for major improvements in the aircraft, motor car and shipbuilding industries and in the design of engine parts and frames. In spite of its wide applications, it is still much in its infancy—or perhaps adolescence—hardly a year passing without a new material or technique being announced.

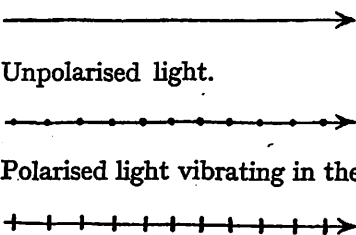
It is the object of this pamphlet to explain for the general interest of engineers the principles of photoelasticity, mainly without proof, which may be found in any of the classical works if desired, and to describe the apparatus and materials necessary and the techniques available to the reader who wishes to carry out his own experiments. An elementary knowledge of the nature of principal stresses and of mathematics on the part of the reader has been assumed.

## II. THE NATURE OF POLARISED LIGHT.

Light consists of vibrations in the ether, believed to be electromagnetic in character, consisting in general of a mixture of wavelengths. The following remarks will be confined to light of any one wavelength known as monochromatic light which may be produced by passing white light through a special coloured filter. The vibrations, assumed sinusoidal in form, occur in all planes which pass through the axis of propagation of the light beam. If a vibration is represented by a vector in the plane of the wave front, which will be taken as the plane perpendicular to the axis of propagation, the vibrations of a normal light beam will be represented by lines radiating in all directions.

Certain filters exist which can resolve these vibrations into any plane so that light emerging from the filter vibrates in only one plane. This light is then said to be *plane polarised* and the filter is known as a *polariser*. The plane through the axis of propagation and at right angles to the plane of the vibrations is known as the *plane of polarisation*, but to save confusion reference will usually be made to the plane of the vibrations. (Fig. 1).

Following normal convention, the following diagrammatic representation will be adopted.

- 
- (a) Unpolarised light.
- (b) Polarised light vibrating in the plane normal to the paper.
- (c) Polarised light vibrating in the plane of the paper.

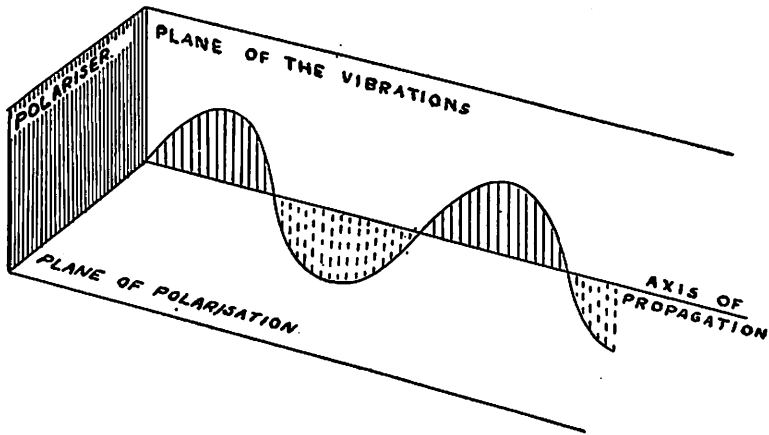


Fig. 1.

### Reflection and Refraction.

Early in the study of polarised light it was found that when ordinary light impinged on a glass plate at an angle  $i$  to the normal, the reflected and refracted rays were, to some extent, polarised in mutually orthogonal directions. (Fig. 2).

The angle  $i$ , at which maximum polarisation occurs, is known as the polarising angle; for glass it is about 57 degrees. OR and OQ are found to be at right angles, so that,

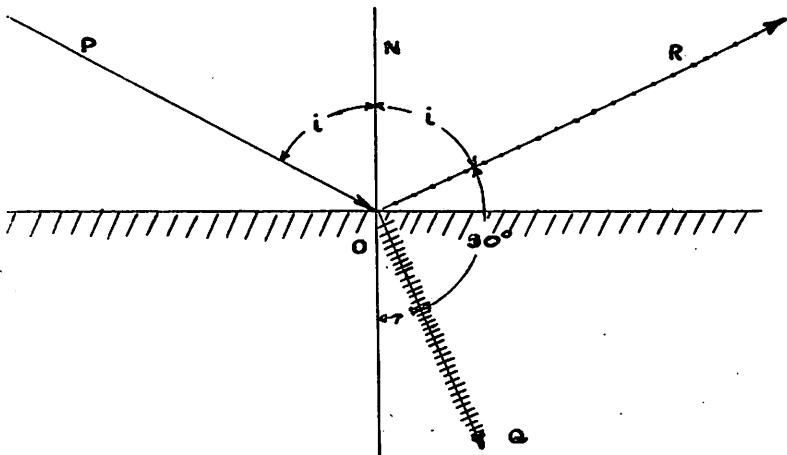


Fig. 2.

$$\begin{aligned} i + r &= \pi/2 \\ \sin r &= \sin (\pi/2 - i) = \cos i. \end{aligned}$$

$$\therefore \text{ the refractive index } n = \frac{\sin i}{\sin r} = \tan i.$$

This is known as Brewster's Law—that the tangent of the polarising angle is equal to the refractive index of the medium. More recently, however, it has been found that this holds strictly for media whose refractive indices are about 1.46.

### Double Refraction.

A beam of light passing through a crystal is split up into two beams polarised in orthogonal planes which pass through the axis of propagation. If, for one particular direction of propagation the two waves travel with equal velocities, the crystal is said to be uniaxial and the axis of propagation is the optical axis. For light passing through a uniaxial crystal in any other direction, the two velocities will be different and the waves will emerge from the crystal out of phase. Glass and some plastic materials achieve this property of birefringence under conditions of stress. The variation in intensity, which is caused by this phase difference of the two polarised waves emerging from a birefringent material, is known as interference and, since the whole of the theory of photoelasticity depends upon the observation of interference, a fuller investigation is demanded.

The observation of this interference is made by resolving the two emergent waves into a plane so that the resultant of the two components may be analysed. In order to observe the interference at all, in fact, the light must be polarised before entry into the birefringent material and analysed in a plane at right angles to the plane of the vibrations of the polariser. The apparatus used is called a polariscope, the simplest form of which is shown in Fig. 3.

### Interference.

Suppose a polarised beam whose vibrations occur in the plane OP (Fig. 4) is represented by  $u = a \sin \omega t$ , where  $a$  is the amplitude. If it passes through a birefringent material it splits up into two orthogonal beams,

$$\begin{aligned} x &= a \sin \omega t \cos \phi \\ \text{and } y &= a \sin \omega t \sin \phi. \end{aligned}$$

As they pass through,  $y$  travels more slowly than  $x$  and becomes retarded relatively by  $\alpha$ , say. They emerge thus as

$$\begin{aligned} x_1 &= a \sin \omega t \cos \phi \\ \text{and } y_1 &= a \sin (\omega t - \alpha) \sin \phi. \end{aligned}$$

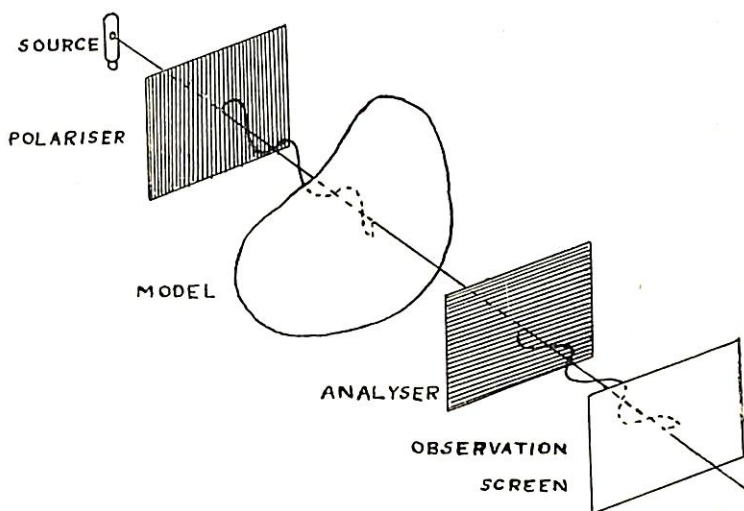


Fig. 3.

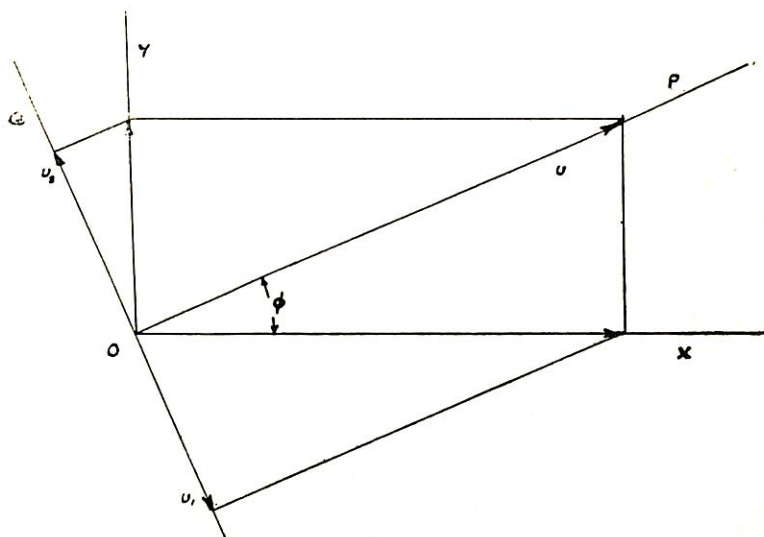


Fig. 4.

Let them now, for examination, be resolved into a plane OQ at right angles to OP; their components will be

$$u_1 = a \sin \omega t \cos \phi \sin \phi$$

$$\text{and } u_2 = a \sin (\omega t - \alpha) \sin \phi \cos \phi.$$

The resultant of these two components will be

$$u_1 - u_2 = a \sin \phi \cos \phi (\sin \omega t - \sin (\omega t - \alpha))$$

$$= a \sin 2\phi \cos (\omega t - \alpha/2) \sin \alpha/2.$$

This is a sinusoidal vibration of amplitude  $a \sin \alpha/2 \sin 2\phi$  and its intensity is proportional to the square of the amplitude. For zero intensity, therefore,

$$\alpha/2 = 0, \pi, 2\pi, \dots, n\pi$$

$$\text{or } 2\phi = 0, \pi, 2\pi, \dots, n\pi$$

*i.e.*, either,

- (i) the original polarised beam must be in one of the planes of double refraction, or
- (ii) the phase difference caused in the material must be an integral number of wavelengths.

These are exactly the two observations of interference made in orthodox photoelastic techniques and will be dealt with more fully in Chapter III.

### Circularly Polarised Light.

Consider the case of two plane polarised beams from the same source, of equal amplitude, emerging from a birefringent material out of phase by  $\alpha$ ,

$$x = a \sin \omega t \quad \text{and } y = a \sin (\omega t - \alpha).$$

Eliminating  $t$ , we have

$$y = a (\sin \omega t \cos \alpha - \cos \omega t \sin \alpha)$$

$$= x \cos \alpha - \sqrt{a^2 - x^2} \sin \alpha$$

$$\therefore (a^2 - x^2) \sin^2 \alpha = y^2 - 2xy \cos \alpha + x^2 \cos^2 \alpha$$

$$\text{i.e., } x^2 - 2xy \cos \alpha + y^2 = a^2 \sin^2 \alpha \text{ which is the equation of an ellipse.}$$

Suppose now  $\alpha = \pi/2$ . The equation then becomes  $x^2 + y^2 = a^2$ , which is that of a circle.

As  $t$  increases from 0 to  $\pi/2\omega$ ,  $x$  increases from 0 to  $a$ , while  $y$  increases from  $-a$  to zero.

The light produced, therefore, by the two waves out of phase by a quarter wavelength is represented by a point rotating in a circular path in an anticlockwise direction (for  $\alpha$  positive). A plate which deliberately introduces this amount of relative retardation in order to produce circularly polarised light is called a quarter wave plate. It should be noted that a given quarter wave plate matches only one particular wavelength and that it is not possible to produce a quarter wave plate which suits white light.

### III. PHOTOELASTICITY.

In 1816 Sir David Brewster observed that glass, when subjected to stress, adopted the properties of a uniaxial crystal, in that as a beam of light was passed through it, double refraction effects were exhibited. Although, with the development of synthetic resins, glass is now regarded as relatively insensitive, it is on the laws formulated by Brewster and later workers, particularly Neumann and Clerk-Maxwell, that the theory of photoelasticity is based.

Light travels with different velocities in different media. It does, further, travel through an unstressed medium with a velocity different from that in the same medium under stress. The difference in phase caused by the passage of a polarised beam through a given thickness of material, stressed and unstressed, is called the absolute retardation. The interference observed in the passage of a beam of light through a birefringent material will be due to the relative retardation of the two orthogonal waves into which the beam will be split. For a parallel plate of a given sensitive material, subjected to a uniform stress  $p$  and into which a beam of light passed normally, it was observed that,

- (i) the beam was polarised into two waves in planes in the direction of  $p$  and at right angles to  $p$ ,
- (ii) the retardation of each wave was proportional to  $p$ ,
- (iii) the retardation of each wave was proportional to the thickness of the material,  $d$ .

Thus for a parallel plate of thickness  $d$ , under a simple tensile stress  $p$ , a beam of light will be split up into two polarised waves along and perpendicular to  $p$ , such that the absolute retardations  $r_1$  and  $r_2$  are given by

$$r_1 = C_1 p d \quad \text{and} \quad r_2 = C_2 p d.$$

The relative retardation therefore is given by

$$r_1 - r_2 = (C_1 - C_2) p d = C p d.$$

$C$  is known as the stress optical coefficient and is constant for a given material up to a particular stress, for a given temperature. Around room temperature  $C$  is fairly constant for most materials.

If now the material is subjected to two stresses,  $p$  and  $q$  at right angles, the retardations due to each stress are additive and

$$\begin{aligned} r_1 &= C_1 p d + C_2 q d & r_2 &= C_2 p d + C_1 q d \\ &= (C_1 p + C_2 q) d & &= (C_2 p + C_1 q) d, \end{aligned}$$

the relative retardation being given by

$$\begin{aligned} r &= r_1 - r_2 = (C_1 - C_2) p d - (C_1 - C_2) q d \\ &= (C_1 - C_2) (p - q) d = C (p - q) d. \end{aligned}$$

The working unit of  $C$ , the Fringe Value  $f$ , is defined on page (36).

### The Laws of Photoelasticity.

The following two empirical laws, forming the basis of photoelasticity, were formulated by Neumann and Clerk-Maxwell and are rather more general than the observations described above.

- (i) At any point in a stressed transparent material the axes of polarisation of light passing through it are parallel to the directions of principal stress in a plane normal to the beam at the point.
- (ii) The difference of the velocities of the two polarised beams at the point is proportional to the difference of the principal stresses at the point and is independent of stresses parallel to the axis of propagation of the light.

For a stressed plate normal to the axis of light propagation, *i.e.* usual two dimensional work, the difference of the velocities is proportional to the relative retardation of the two waves through the plate.

### Observations of Interference.

Consider now a simple polariscope in which a stressed parallel plate is under examination. The axis of the polariser is horizontal and that of the analyser vertical while the major principal stress  $p$  is in  $Ox$  at an angle  $\phi$  to  $OP$ . (Fig. 5).

Let the vibrations emerging from the polariser in plane  $OP$  be represented by

$u = a \sin 2\pi/\lambda (Vt - z)$ , where  $u$  is the displacement at time  $t$  and  $\lambda$  the wavelength.

As this polarised wave passes into the stressed material it is resolved into the planes of principal stress  $Ox$  and  $Oy$  with components

$$x = u \cos \phi \quad \text{and} \quad y = u \sin \phi.$$

During the passage through the stressed plate they will be retarded relative to one another by an amount  $r$ , given by

$$r = C (p - q) d.$$

As they emerge therefore they will be given by

$$x_1 = a \cos \phi \sin 2\pi/\lambda (Vt - z - r).$$

$$y_1 = a \sin \phi \sin 2\pi/\lambda (Vt - z).$$

In order to observe the interference caused by the phase difference the two waves are now resolved into a vertical direction by the analyser, giving components

$$u_2 = y_1 \cos \phi \quad u_1 = x_1 \sin \phi,$$

the resolute of which will be

$$\begin{aligned} u_2 - u_1 &= a \sin \phi \cos \phi \sin 2\pi/\lambda (Vt - z) - a \cos \phi \sin \phi \\ &\quad \sin 2\pi/\lambda (Vt - z - r). \\ &= a \sin \phi \cos \phi \{ \sin 2\pi/\lambda (Vt - z) - \sin 2\pi/\lambda (Vt - z - r) \} \\ &= a \sin 2\phi \sin \pi r/\lambda \cos 2\pi/\lambda (Vt - z - r/2). \end{aligned}$$

This is a vibration whose amplitude is  $a \sin 2\phi \sin \pi r/\lambda$ .

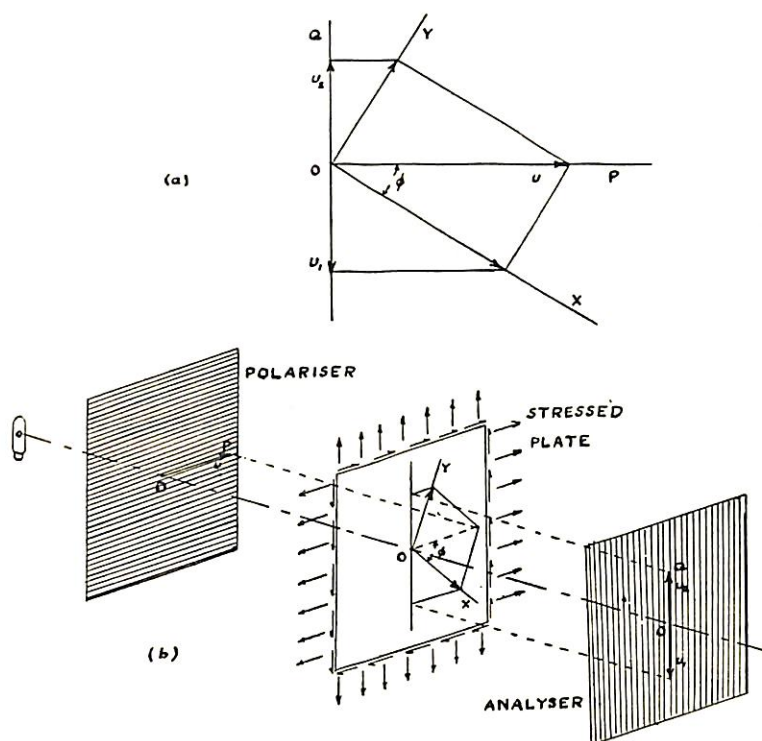


Fig. 5.

Since the intensity of light is proportional to the square of the amplitude, the intensity of the observed resolute is proportional to  $(a \sin 2\phi \sin \pi r/\lambda)^2$

The observed intensity will be zero, therefore, at the point under observation if,

- (i)  $2\phi = 0, \pi, 2\pi \dots n\pi$  where  $n$  is an integer,  
 or (ii)  $\pi r/\lambda = 0, \pi, 2\pi \dots n\pi$ , i.e. if  $r = C(p-q)d = 0, \lambda \dots n\lambda$ .

### The Isochromatic Fringes.

Taking condition (ii) first, the relative retardation for zero intensity is

$$r = C(p-q)d = 0, \lambda, \dots n\lambda.$$

At all points where the relative retardation is zero dark spots will occur, forming in general a dark line. Similar dark lines will appear forming the locus of points where the relative retardation is one wavelength, that is to say, the stress difference is of a constant

value causing a difference of phase of one wavelength. Similarly for two wavelengths retardation, and so on. These dark lines are called the isochromatic fringes, or simply fringes, defined as the loci of points at which the principal stress difference is the same. Depending on the number of wavelengths relative retardation, the isochromatic fringes are known as the zero order fringe, first order fringe, etc.

A source of white light will emit the whole visible spectrum. The wavelengths will vary between 3,900 and 7,600 units, representing violet to red light. At a point where the stress difference is such that the relative retardation is 3,900 units, the violet light will be extinguished and the remaining colours cause yellow to be observed. As the relative retardation increases, bands of red, blue and green will be seen, corresponding to the extinction of blue, green and red. At larger retardations, several colours are extinguished together and the bands become fainter and less definite. The principal use to which the engineer puts these colour bands is in the change from red to blue, which is exceedingly sensitive to small changes in relative retardation. Use is made of this sensitivity when a uniform stress field is required, lack of uniformity being immediately apparent by red or blue tinges.

Most light sources are a mixture of several wavelengths, so that when an examination of the isochromatic fringes is being made, a monochromatic filter must be used, producing clear black lines in the picture. If the ground glass screen of a plate camera is being used to examine the picture, this filter may conveniently be placed between the screen and the camera lens.

In order to distinguish the orders of the fringes in a stressed plate, it is necessary to watch their appearance as the load is slowly applied. The fringes will emanate from the points of highest stress difference and by counting the fringes as they move out, the orders of the fringes at these points may be found when the whole load has been applied. If it is still not clear in which direction the fringe order is increasing, the filter should be removed, allowing the colours to be observed. The colour changes from red to blue in the direction in which the fringe order increases. The whole picture of the isochromatic fringes appearing in a plate under stress is called the fringe pattern.

### **The Isoclinic Lines.**

The first condition for zero intensity,

$$\phi = 0, \pi/2, \dots n\pi/2,$$

is the condition that the directions of the principal stresses should be along the axes of polariser and analyser. At every point in the model where the principal stresses are parallel to polariser and analyser axes, a dark spot will appear. These spots will, in general,

form a dark line across the picture of the model, called an isoclinic line or simply an isoclinic. Since the directions of principal stress will, in general, change from point to point, for different inclinations of polariser and analyser the lines will be different, each named the 'N' degree isoclinic for a rotation of the polariser 'N' degrees from the vertical (conventionally in an anticlockwise direction looking towards the source). It will be noticed that the condition does not depend on the wavelength, so that the isoclinics appear always the same dark colour which the light source shows through crossed polarising media. The sharpness of the isoclinics will depend on how quickly the directions of principal stress change across the plate and in the case of a wide band, the line of deepest intensity should be taken. In some cases, of course, such as a uniform bar in tension, the whole of the middle of the bar will be uniformly dark. Normally, it will be found convenient to plot the isoclinics, if they should be required, at a fairly low stress when they are clearer and before they become confused by the isochromatic fringes.

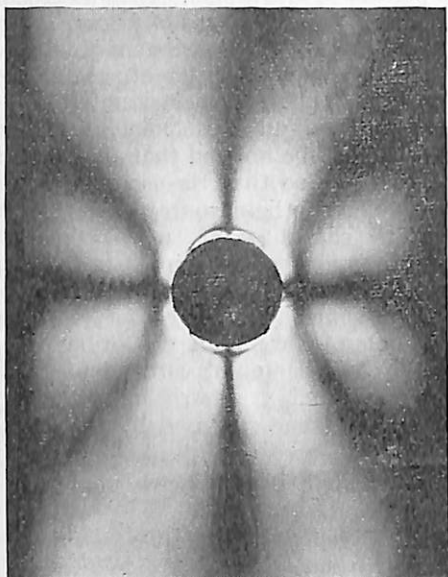
### Isotropic Points.

With many problems, it will be found that the isoclinics pass through one or more fixed points as the polariser and analyser are rotated. These isotropic points, as they are called, are points in the model at which the stress is the same in all directions, behaving like a source of uniform tension or compression emanating radially. Such uniform tension or compression can, of course, be zero. Because the stress difference at isotropic points remains zero for any load, they appear as black spots in the fringe pattern, unaltered in position as the load increases. Usually, isotropic points are isolated but occasionally they may be joined to form what is called a neutral line. A neutral line for example, occurs along the neutral axis of a beam in pure bending. There are two distinct and important types of isotropic points.

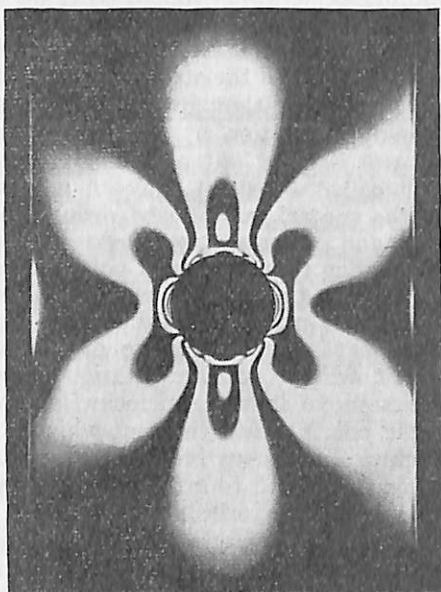
Regarding the rotation of polariser and analyser together in an anticlockwise direction as positive, it will be observed that the isoclinic lines may rotate about the isotropic points in either a clockwise or an anticlockwise direction. Those isotropic points at which, with a positive rotation of polariser and analyser, the isoclinics move in an anticlockwise direction are called positive isotropic points, while those at which the motion of the isoclinics is clockwise are negative isotropic points. The two kinds of isotropic point lead to a fundamental difference in the orientation of the stress trajectories.

### Stress Trajectories.

Principal stress trajectories, or more simply, stress trajectories are defined as curves which at any point are tangential to one of



(a) The Isoclinics at Zero Degrees.



(b) The Isochromatics.

**Fig. 6**—A hole in a Uniform Tensile Strip.

the principal stresses at that point. They are also known as isostatics or lines of principal stress. Two sets of lines can obviously be drawn, representing the major and minor principal stresses and these two sets of lines must always cut each other at right angles. They must further enter a free or unstressed edge normally. By drawing light lines at points along each isoclinic parallel to the axes of polariser and analyser at the relevant angle of inclination, the stress trajectories may readily be drawn. If greater accuracy is required, geometrical constructions, described fully in the references, should be used. Isotropic points, however, will give rise to some difficulty.

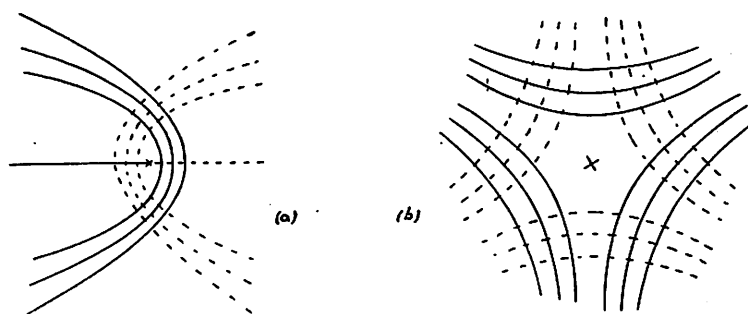


Fig. 7.

It can be shown that for a positive isotropic point, the two sets of stress trajectories must interlock as shown in Fig. 7a, while for a negative isotropic point, the stress trajectories will avoid one another in the same way as the lines of magnetic force bypass a neutral point in a magnetic field. (Fig. 7b).

The plate under examination may therefore be completely mapped in stress trajectories and the directions of the principal stresses at any point may be found by interpolation. The isoclinics and stress trajectories are shown for a simply supported beam with a concentrated load at the middle in Fig. 8.

### Application of Results.

At this point, it is as well to pause and summarise what results may be obtained from the preceding simple observations made in orthodox two dimensional work before proceeding to a description of the various techniques available for a thorough stress analysis of every problem. Frequently it will be found that a particular problem may be solved by a simple examination of isoclinics and isochromatics, even though this examination yields limited information.

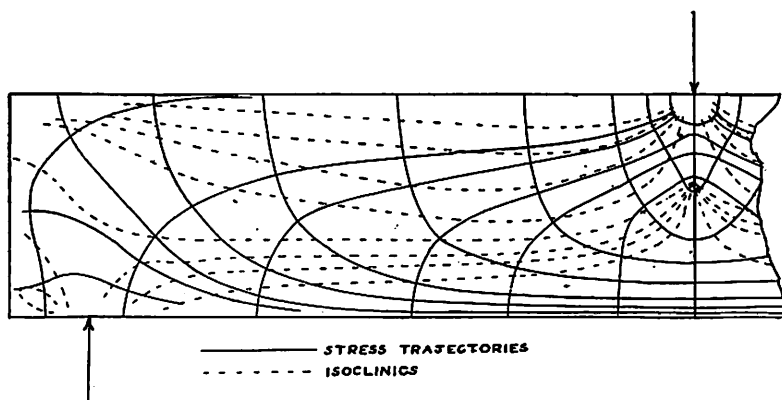


Fig. 8.

- (i) From the isoclinics and isotropic points the stress distribution over the whole plate may be plotted. The picture of the flow of lines of principal stresses gives a very general idea of the efficiency of the structure. If, for example, it is possible to draw the lines of principal stress equidistant across a section of the plate which is under uniform tension, the distance apart of the lines at other points and the general flow of the lines will show what parts are contributing most to transmitting the stress, and what parts are merely "deadwood." The isoclinics themselves also yield useful guidance as to whether the model is loaded correctly, for example if a uniform tension field is being applied, or if the model is behaving symmetrically about an axis of physical symmetry. They will, further, show if the model is entirely stress free before the load is applied because they appear at a low stress well before a whole fringe appears. Indeed, this is the very use to which a simple polariscope is put by plastics manufacturers to ensure that their products leave the factory stress free—an important consideration, for example, with plastic television screens or optical equipment.
- (ii) The fringe pattern will show the difference of the principal stresses at any point in the plate. This, frequently, is all that is required because the maximum shear stress at a point is equal to one half the principal stress difference. Should the model be that of a mild steel structure it will be the maximum shear stress which will determine failure, but it should be emphasized that a higher shear stress might be obtained by a combination of the major principal stress and the zero stress normal to the plate. Points

to observe, however, will be indicated should a steel model be built subsequently and coated with plumbers' resin in order to detect yield. Knowledge of the shear stress along a plane throughout the model is sometimes desirable for a condition of equilibrium, as in the case of a load diffused into a plate by shear from a stringer. Since the shear stress in a plane at an angle  $\theta$  to the major principal stress is given by  $\frac{1}{2} (p-q) \sin 2\theta$ , both isoclinics and fringes would be required in this case.

- (iii) Stress concentrations most frequently appear at free edges in a plate. Since there can be no normal stress at a free edge, the principal stress difference indicated by the fringe order is the actual value of the tangential stress at the point. Often, therefore, at the place where it is most vital to the safety of a structure, the stress is indicated directly. It can be readily verified, for example, that the circumferential stress at the sides of a small hole in a uniform tensile strip is, as given by theory, three times the mean stress in the member remote from the hole. (See Fig. 6).
- (iv) Among the many other applications which may be relevant to particular problems, one further one will be mentioned. By observing the neutral lines, the zero order fringes and parts of the model not contributing to any extent to the strength, positions of holes or "stress raisers" may be chosen which will not greatly influence either the stress distribution or the strength of the model.

### Limitations of Elementary Practice.

Although the elementary observations just discussed yield a wealth of information which it is not possible to obtain in any other way, it is as well to formulate their limitations in order that the means by which these are overcome may be approached logically in succeeding chapters. It has not so far been possible :—

- (i) To obtain  $p$  and  $q$  separately except at the free edges of a model.
- (ii) To deal with a three dimensional model. Only the results for a plate subjected to stresses in the plane normal to the direction of light propagation have been explained.
- (iii) To assess the stress difference between integral order fringes except by interpolation, which may be of doubtful accuracy especially if the fringes are wide apart. Fractional order fringes are discussed under "Use of Compensators."

- (iv) To select the best technique for the quickest and most accurate solution of a particular problem.
- (v) To compare the materials and type of equipment which can be used, knowing the limitations of each and the errors which may arise.
- (vi) To use any mathematical theory which will guide the investigator. The mathematics involved is rather advanced and severely limited in application and it is not proposed in this short pamphlet to discuss it. The classical references will supply such information if the reader desires.

#### IV. EXPERIMENTAL TECHNIQUES.

This chapter is not intended to be a mere description of apparatus. Where new apparatus is involved a description of it is given but the intention is to introduce new principles or extend those already presented as brought out by the various techniques which may be employed for the solution of different problems.

##### The Plane Polariscope.

This has already been discussed to some extent but the details of experimental apparatus remain to be described. The simplest form of plane polariscope comprises a diffused light source covered with a sheet of Polaroid, a space in which the model is held for examination and a crossed sheet of Polaroid, the analyser, through which the model is observed. A polariscope of this type is used for little more than demonstration purposes or for the examination of manufactured plastic articles to ensure absence of initial stress. For the modest experimenter the simplest type, which can be readily made in the workshop, is shown in Fig. 9.

A nest of bulbs comprises the light source S, the light from which passes through a cooling water cell W, to a diffusing screen D. The diffusing screen may be ground glass or a sheet of Alkathene protected by glass. P is the polariser and A the analyser while

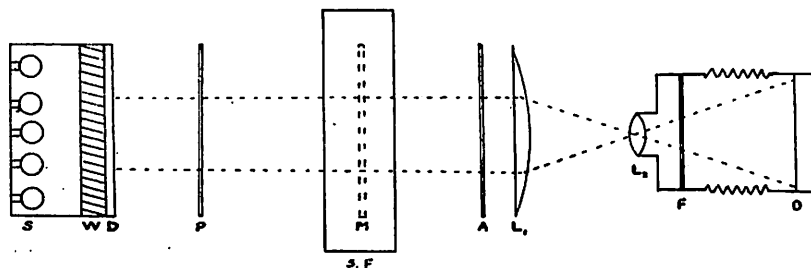


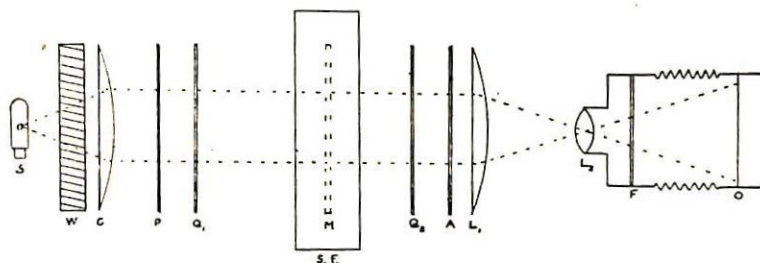
Fig. 9.

between them is the model mounted in some sort of straining frame which may be one of many designs. Often it comprises a stout framework from which the load is applied by screw, the load being measured by the deflection of a beam or spring on which the screw is mounted. Usually the bench mounts a permanent means by which a vertical load is applied and a portable frame for applying loads in other directions. The vertical load may be applied hydraulically or, more simply, by weights which are supported through rubber on a platform which can be lowered away from them slowly. A lens,  $L_1$ , concentrates the rays into the camera lens,  $L_2$ , through a monochromatic filter,  $F$ , to the observation screen,  $O$ , which may, of course, be replaced by a photographic plate or a sheet of tracing paper for sketching. It is a great advantage if the polariser and analyser can be rotated together from the observation position, but in any case both should be calibrated in degrees relative to the vertical. There may be many variations of the set-up described, of course, particularly with the viewing arrangements and light source. The former may consist of a large screen on which the picture is thrown, or the picture may be merely observed without  $L_1$  and the camera. For the illumination, a compact light source, as described for the circular polariscope, gives a clearer picture and more light.

However, with a plane polariscope all the observations described heretofore can be made, forming the basis of photoelastic research. All other techniques, with the exception of the use of scattered light, are variations of it. The drawback of the plane polariscope is the confusion which arises in observing the isoclinics and isochromatics together but this disadvantage is overcome in the circular polariscope.

### The Circular Polariscope.

The circular polariscope is precisely the same as the plane instrument but with the insertion of two quarter wave plates. To cover as much ground as possible, however, it has been drawn in Fig. 10 with a compact source lamp,  $S$ , which requires a com-



10.

compensating lens, C, for the production of parallel light through the model and polariser. The compact source lamp consists of a mercury arc across silver electrodes, giving a high illumination over a very small source which makes it excellent for the optical bench. The two quarter wave plates,  $Q_1$  and  $Q_2$ , are inserted between the model and the Polaroids as shown. They are each rotatable over a graduated scale and portable so that they may be swung out of position transforming the circular polariscope into a plane type.

It has previously been shown that the insertion of quarter wave plates produces circularly polarised light through the model. The object of this is to eliminate the isoclinics and allow free observation of the isochromatic fringes. The reason for the elimination of the isoclinic lines is best seen by mathematics, which is unfortunately somewhat lengthy for inclusion in this pamphlet, but a general idea can be explained.

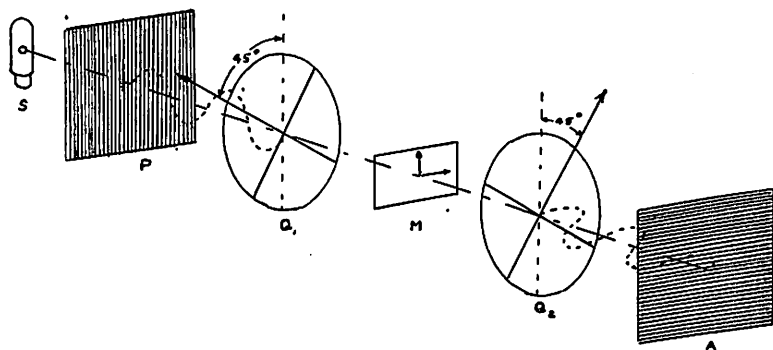


Fig. 11.

It will be remembered that an isoclinic is produced when a line of principal stress is parallel to the axis of the polariser. The vertical vibrations pass through the model without resolution and are stopped completely by the horizontal analyser. If  $Q_1$  is introduced, the vertical vibrations are split into two components at 45 degrees to the vertical out of phase by a quarter wavelength. As these two waves pass through the model, horizontal and vertical components emerge which are split into the same planes by  $Q_2$  as the original vibration was by  $Q_1$ . This time, however, they are shifted in phase in the other direction, compensating the quarter wavelength shift caused by  $Q_1$ . The two waves at 45 degrees to the vertical which emerge from  $Q_2$  will have a horizontal component which will pass through the analyser and no light will be extinguished due to a line of principal stress being parallel to the polariser axis.

There is a different arrangement which can prove quite valuable. If, instead of the two quarter wave plate axes being crossed, they are made parallel, still at 45 degrees to the polariser and analyser axes, the quarter wavelength relative retardation caused by  $Q_1$  is no longer compensated by  $Q_2$ . At every point a relative retardation of one-half a wavelength is superimposed and the isochromatic fringes will represent retardations of  $1/2 \lambda$ ,  $3/2 \lambda$  . . .  $(n + \frac{1}{2}) \lambda$ . Thus, with a circular polariscope, the isoclinics are eliminated, integral order fringes are observed when the quarter wave plates are crossed and half order fringes are observed when the quarter wave plate axes are parallel. The "mixed set-up," as it is called, producing half order fringes, gives a light background. A similar effect is obtained by crossing the quarter wave plates and making the polariser and analyser axes parallel, but each of these methods suffers from the disadvantage that small errors will add, causing the fringes to be misplaced slightly. There is a further service which may be rendered by the quarter wave plates, discussed under "Use of Compensators."

### Use of Compensators.

It is sometimes necessary to use materials which are relatively insensitive or materials of such thickness that only one or two fringes become apparent at the desired load. In such cases, fractional fringes may be measured by one of the forms of compensator.

**The Coker Compensator.**—A strip of the same material as that of the model is mounted in a frame which can apply a known tension,  $T$ , to the strip. If the fractional fringe is required at a point at which the principal stresses are  $p$  and  $q$  ( $p$  greater than  $q$ ), the tension strip is inserted in the polariscope with its centre in the line of vision and loading axis parallel to the direction of  $q$ . Tension is then applied until the light emerging from the analyser at the point is a minimum. If the width of the tension strip is  $b$  and its thickness is  $d$ , the value of the stress difference at the point is given by

$$p - q = T/bd$$

**The Soleil-Babinet Compensator.**—In this case, the superimposed relative retardations are brought about by two thicknesses of quartz with their optical axes crossed. The thickness of one quartz block is varied by dividing it into two quartz wedges, sliding one over the other so that the relative retardation alters accordingly. The instrument is placed in the polariscope close to the model with its axes parallel to the directions of principal stress at a point. The compensator is then adjusted so that the zero order fringe is seen through the analyser; the compensator

scale and calibration curve shows what superimposed relative retardation was necessary to compensate that occurring in the model.

**Quarter Wave Plate Used as a Compensator.**—Introduced by Tardy, this is a very useful method because it does not involve the purchase of expensive equipment in addition to the polariscope. It is simple and accurate. The first quarter wave plate, that nearer the lamp, is not required. Polariser and analyser are crossed to make 45 degrees with the directions of principal stress at the point under observation and the quarter wave plate between the model and the analyser is placed with its axes in the same directions. The analyser is then rotated until the light at the point is a minimum, when it can be shown that the relative retardation at the point is  $(n \pm \theta/180)$  wavelengths, where  $\theta$  is the angle through which the analyser is turned. A brief inspection of the fringes will determine the value of  $n$  and the relevant sign. A similar and equally convenient method is available in which analyser and quarter wave plate are rotated together.

### The Portable Polariscope.

A useful technique, particularly in the examination of ship or aircraft structures, involves the use of a small portable polariscope in conjunction with complicated Xylonite models. Fig. 12 shows a section of the instrument.

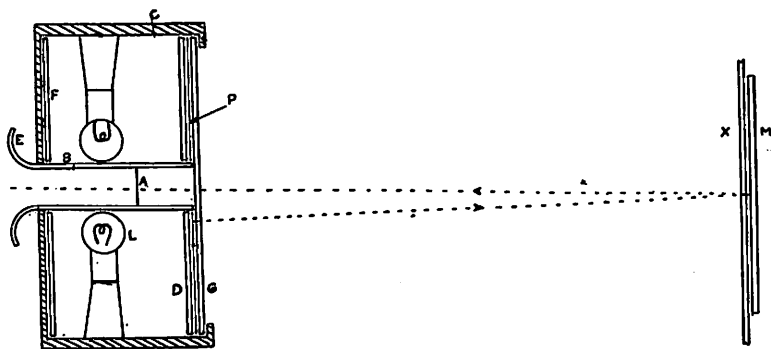


Fig. 12.

A small cylindrical case, C, houses an annular lamp source, L, backed by a mirror, F. D is a disc of a diffusing material, P is the polariser and G, a protecting glass. Surrounding the axis of the instrument is a metal tube, B, ending in an eyepiece which houses the analyser, A. The line of polarisation is marked on the protecting glass.

The plastic model under examination is backed by mirrors and the polariscope is held close to the eye at some distance from the model. The light passes from the source and diffuser, through the polariser, into the stressed model. It is reflected by the mirror, *M*, passes through the birefringent plastic a second time, making a double thickness effective, and into the central tube where the interference is observed through the crossed analyser. When the stress in the model is parallel to the axis of polarisation, the light will, of course, be a minimum and the direction of principal stress may be marked on the model as indicated by the line on the protecting glass. By moving over the whole model in this way, the stress trajectories may be plotted with high accuracy.

Plastics which can be built into highly complex structures, such as Xylonite, are relatively insensitive photoelastically so that fringes will not appear. In order to find the stresses in the model, wire resistance strain gauges may be stuck on the Xylonite, oriented along the lines of principal stress, where desired, obviating the use of strain gauge rosettes and saving, in complex models, an enormous amount of analysis. Up to now, however, due to the creep in Xylonite and movement between gauge and plastic, it has not been possible to relate with any accuracy these stresses to the stresses in the ship or aircraft. The tests have been comparative; that is to say, with slight modifications to the model, the stresses have been reduced to a minimum and the best model selected by which to design the structure.

Here, however, there is room for research, for there is no reason why the stresses measured at a given time after loading should not be related in some way to the stresses at the time of loading, avoiding thus the effects of creep. The stresses deduced for zero time could then be related to the stresses which occur in the full scale steel structure. There are further complications. With thin sheets of plastic a strain gauge may cause local stiffening which, in some cases, may affect the reading. Temperature and humidity have marked effects on the behaviour of the material and constancy of these parameters is highly desirable during manufacture and test of the models.

### Three Dimensional Photoelasticity.

Excepting, for the moment, the use of scattered light, which is discussed later, there is only one general technique available for the examination of stress in a three dimensional model. As a beam of light passes through such a model, the directions of the three principal stresses will, in general, change from point to point, so that any interference observed by using polariser and analyser will yield no information about the stress at any particular point. It will be remembered that one of the requirements for the ex-

amination of a stressed lamina, is that the magnitude and direction of the stress should not change through the plate.

Some of the photoelastic materials, when heated to a softening temperature, exhibit a pattern when stressed and examined in the polariscope similar to that of cold models under load. Moreover, this stress pattern is maintained if the model is allowed to cool under load and the load then removed. Since the stress pattern is, in effect, frozen into the material, this method is known as the frozen stress technique. It is found that if the stress frozen model is cut or machined carefully, the stress pattern is unaltered and a thin lamina cut from it exhibits the stress pattern relevant to the state of stress in that plane in the uncut model. There are severe limitations, however, which must be emphasized. A plate cut at random from a frozen model will exhibit the usual features in a polariscope, but the two principal stresses observed will not necessarily be two of the three principal stresses occurring in the three dimensional model. Indeed, the chance of selecting a plane which contains two of the principal stresses is very remote without other knowledge and, in any case, the directions of the three principal stresses will, in general, change from point to point in the plane. In fact there are only a few planes in any model in which two of the three principal stresses always lie. They are:—

- (i) Unloaded surfaces over which the normal stress must everywhere be zero.
- (ii) Planes of symmetry.

Of course, for the condition of the stress to be sensibly constant through the thickness of the plate, a requirement which must be met, thin slices must be cut. Although limited thus to planes of symmetry and surfaces, a knowledge of the principal stress difference there is often valuable since it is most frequently at free surfaces that maximum stress differences and stress concentrations occur. The maximum stress difference, however, may not be given by an examination of the slice; it may be due to a combination of the zero stress,  $r$ , normal to the surface and one of the principal stresses,  $p$  or  $q$ , in the plane of the slice.

To determine the value of the maximum of the three principal stress differences, a second slice must be taken containing  $p$  and  $r$ , such as that shown in Fig. 13. From the second slice,  $p - r$  is found, and since  $r$  is zero, the separate values of the principal stresses,  $p$  and  $q$ , are determined. Usually, the first model will be mutilated and it will be necessary to make two exactly similar models. From the first, a slice is cut off the surface, finding the directions of  $p$  and  $q$  and the value of  $p - q$  at any point, while from the second slice, the actual value of  $p$  is determined. Thus, it has been shown that the values and directions of the principal stresses at any point on the free surface of a three dimensional

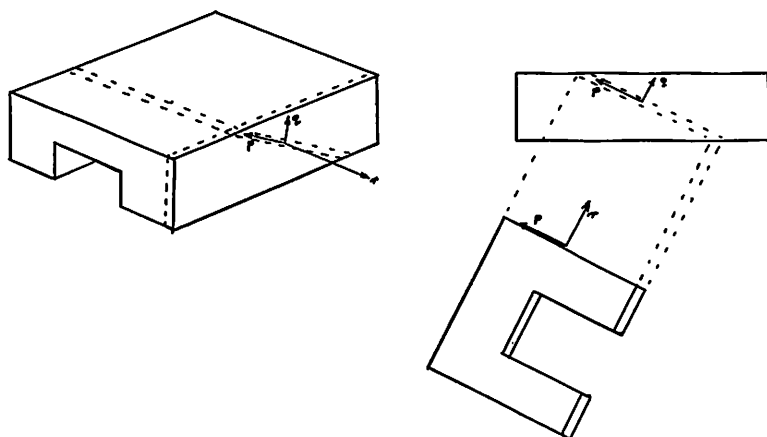


Fig. 13.

model can be found. The case of the plane of symmetry is not so fortunate.

A plane which is symmetrical with respect to both the loading and the geometrical shape of the model, will contain two of the three principal stresses existing at any point on the plane. The third principal stress will be everywhere normal to the plane, but it will not, in general, be zero. Such a plane, therefore, may be cut out as a slice and examined in a polariscope but extra care must be exercised with regard to the thickness. At stress concentrations, particularly, the directions and magnitudes of the stresses may not be constant through the material and the reliance which may be placed on the fringe pattern should be judged by cutting two or three such slices of varying thickness close to the concentration.

The particular case of the plane of symmetry containing an axis of symmetry occurs often and is worthy of special mention. Fig. 14 shows the stress trajectories in a diametral slice of a sphere under compression. The problem may be treated quite satisfactorily by the frozen stress technique until the points A and B are reached when, even with the thinnest slice, the stress directions change rapidly through the slice and the fringes can no longer be relied upon. Since at any point on AB one principal stress must always be vertical from reasons of symmetry, horizontal slices can be taken to find the maximum stress difference at any point on the axes of models such as these, except near stress concentrations.

Summarising the capabilities of the frozen stress technique :—

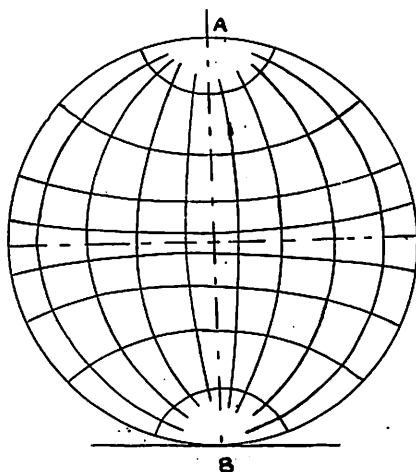


Fig. 14.

- (i) The magnitudes and directions of the principal stresses at any point on the free surface of a three dimensional model may be found.
- (ii) The stress directions and stress difference magnitudes may be found for any point in a plane of geometrical and stress symmetry, care being necessary near stress concentrations.

### The Scattered Light Technique.

This valuable technique has very recently been developed for general use by Colonel H. T. Jessop of University College, London, and may well take its place in all well-equipped photoelastic laboratories. Unfortunately, the length of this pamphlet will not allow a full description of the technique but all readers are recommended to obtain the reference quoted in the bibliography.

A beam of sunlight can be seen from the side because of light which is scattered in all directions by tiny particles in the air. A beam of polarised light passing through a doubly refracting material is subjected to interference caused by the relative phase displacement. Observation of this interference can be made in a plane through the point normal to the direction of propagation. This interference will be of a somewhat different nature from that observed in the direction of propagation.

Let the displacements of the two polarised waves at any point in the material be

$$u = a \cos nt \text{ in direction OA}$$

and  $v = b \cos (nt + \psi) \text{ in direction OB. (See Fig. 15).}$

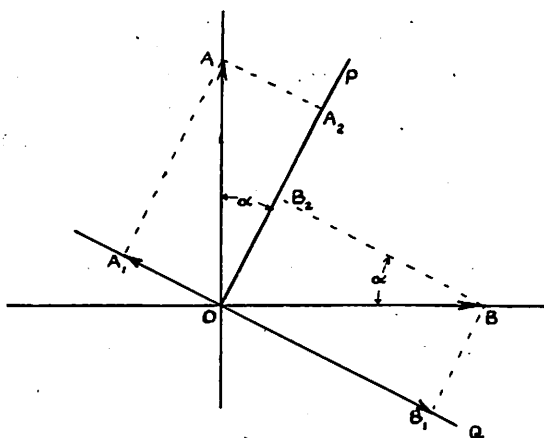


Fig. 15.

Viewed from a direction OP, these two waves will appear as

$$u_1 = a \sigma \cos nt \sin \alpha$$

$$\text{and } v_1 = b \sigma \cos (nt + \psi) \cos \alpha$$

where  $\sigma$  is a coefficient of scattering for the medium.

The resultant observation therefore is  $v_1 - u_1 = \sigma (a^2 \sin^2 \alpha + b^2 \cos^2 \alpha - 2ab \cos \alpha \sin \alpha \cos \psi)^{\frac{1}{2}} \cos (nt + \beta)$ , which is a vibration whose amplitude varies from  $\sigma (a \sin \alpha - b \cos \alpha)$  when  $\psi = 0$  or  $2r\pi$  to  $\sigma (a \sin \alpha + b \cos \alpha)$  when  $\psi = (2r-1)\pi$ , where  $r$  is an integer.

Thus, the light intensity, proportional to the square of the amplitude, will vary along the beam for a given direction of observation. The light intensity variation is accentuated if the direction of observation is varied because if  $\alpha = \tan^{-1} a/b$  the bright fringes will be brightest and if  $\alpha = \tan^{-1} b/a$  the dark fringes will be black. The interference observed will be related to the principal stresses existing at the point. It remains to find this relationship for the two obvious cases which arise :—

- (i) The case in which the principal stresses do not change direction along the path of the beam.
- (ii) The case in which the principal stresses change in direction from point to point along the beam, rotating together, as it were, about the axis of propagation.

**Case (i).** By the laws of photoelasticity, the rate of change of phase with respect to distance of one polarised wave relative to the other is proportional to the difference of the principal stresses (in this case the secondary principal stresses) at any point

$$\text{i.e., } p^1 - q^1 = C \frac{d\psi}{dx}.$$

If then, a graph is plotted of interference fringes against distance, the slope at any point gives the value of  $\frac{d\psi}{dx}$  there and therefore the secondary principal stress difference.

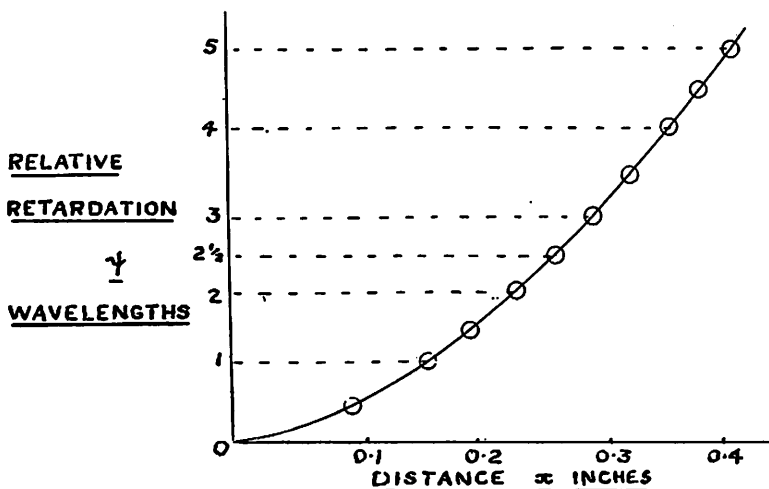


Fig. 18.

**Case (ii).** In this case the directions of the secondary principal stresses change smoothly along the path of the beam. The interference observed from a given direction will depend upon the stress difference and the rotation of the axes. If the two waves at a point,  $x$ , and time,  $t$ , are  $u = a \cos nt$  and  $v = b \cos (nt + \psi)$ ,  $\phi$  is the phase difference attributable to the stress difference and  $\alpha$  the angle through which the stress axes rotate, it can be shown that

$$\frac{d\psi}{dx} = \frac{d\phi}{dx} + (a/b - b/a) \sin \psi \frac{d\alpha}{dx}.$$

The required rate of change of phase difference is  $\frac{d\phi}{dx}$  while that observed is  $\frac{d\psi}{dx}$ . The rate of change of the direction of the axes  $\frac{d\alpha}{dx}$  will, in general, be unknown, but it will be observed

that the term involving  $\frac{d\alpha}{dx}$  is zero if either  $a = b$  or  $\sin \psi = 0$ .

The first condition can be satisfied by arranging the directions of the principal stresses to be at 45 degrees to the plane of incident vibrations but this holds only for the point of entry into the model.

At full and half fringes, the second condition is satisfied but the value of  $\frac{d\psi}{dx}$  at these points cannot be obtained by plotting

a smooth curve through the observed points because the curve is not smooth—it is a series of humps and hollows. It can be shown, however, that if the fringes appear sharp and definite along the beam, the error involved in drawing a smooth curve is small and

$\frac{d\psi}{dx}$  may be used to find the secondary principal stress difference at full and half fringe points. It can further be shown that the larger the model, the smaller the error.

**Application of Theory.** A narrow, intense beam of polarised light is required for this technique, and a suggested optical arrangement is shown in Fig. 17. A compensator introduced in the parallel beam will give fractional fringes. The examining beam must be as parallel as possible.

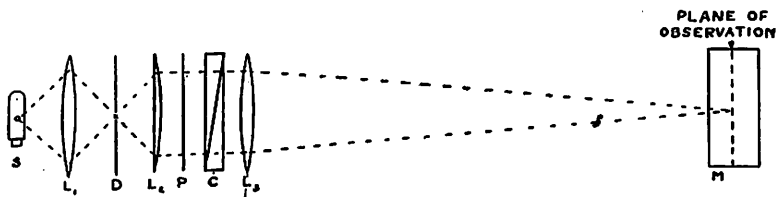


Fig. 17.

There are four types of problem in which the technique may be readily employed, the first one of which is described in detail :—

- (i) Determination of  $p + q$  in two dimensional models.
- (ii) Determination of the secondary  $p - q$  in a plane of symmetry in a three dimensional model.
- (iii) Determination of  $p$  and  $q$  in a prismatic bar under pure torsion.
- (iv) Determination of  $p - q$  at a free surface in a three dimensional model.

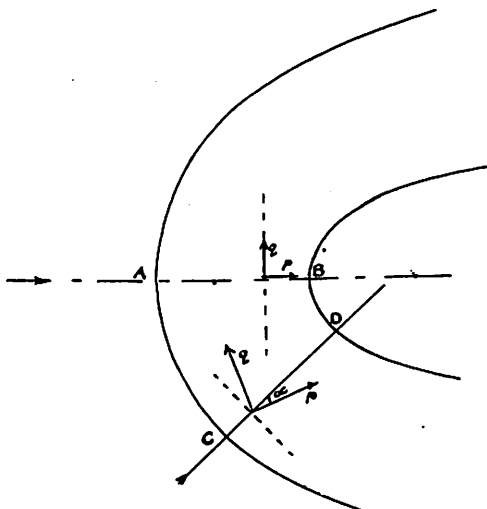


Fig. 18.

The plane of observation in a two dimensional model will contain only one secondary principal stress, the other, normal to the surface being zero. If the beam passes along a line of symmetry such as AB (Fig. 18), this stress will be one of the principal stresses  $q$ . Along any line such as CD, the stress at a point in terms of the principal stresses existing will, from a knowledge of the nature of principal stresses, be  $p \sin^2 \alpha + q \cos^2 \alpha$ . From a circular polariscope  $p - q$  may be found so that the complete state of stress existing throughout the model may be found accurately and simply.

## V. DETERMINATION OF THE SEPARATE STRESSES.

It has been shown already that the separate stresses  $p$  and  $q$  can be found at free edges and surfaces. The determination of  $p + q$  leading with a knowledge of  $p - q$  to the determination of the separate stresses has given rise to many ingenious methods, most of which require great care and accuracy if the result is to be relied upon. Possibly the most powerful of the methods, that using scattered light, has already been briefly described.

### Lateral Extensometers.

It is well-known that the strain in the lateral direction at a point in a lamina where the principal stresses are  $p$  and  $q$  is given by

$$e = \frac{p+q}{\sigma E}, \quad \text{where } \sigma = \text{Poisson's Ratio} \\ \text{and } E = \text{Young's Modulus.}$$

The measurement of this strain is an obvious way of determining  $p+q$ . If  $\frac{1}{4}$ " Catalin 800 is the material in use,  $\sigma = 0.35$  and  $E = 0.19 \times 10^6$  lbs/in.<sup>2</sup> The first order fringe at an edge represents about 200 lbs/in.<sup>2</sup> stress, giving a lateral compression of about 0.00075", which is a very small quantity to measure with any accuracy. Several types of extensometers exist for the measurement of such small movements.

The mechanical type usually follows the pattern used by Coker. This was a caliper style instrument, the movement of the jaws being transmitted through a bell crank lever, rotating a mirror. A long beam of light reflected in the mirror, moved over a graduated scale, giving a high magnification of the movement. The electrical extensometer of the caliper type relies upon the change in resistance of wire resistance strain gauges placed round the bottom of the U, while the jaws follow the movement of the plastic as the load is put on.

Probably the most accurate kind of lateral extensometer is the interferometer type, which can record changes in thickness of the order of a millionth of an inch. It is dependent upon the fact that as a beam of light passes normally into a glass plate, some is transmitted and some is reflected, and that a retardation of a half wavelength occurs due to the reflection. A beam of light starting from O (Fig. 19), for example, partially reflected from the lower surface of A, and the upper surface of B, will result in a beam, shown dotted, in which interference depending on the thickness of the air gap,  $h$ , is observed. It can be shown, in fact, that interference will occur every time  $h = 0, \frac{1}{2}, 1 \dots n/2$  wavelengths of monochromatic light. Thus, a measuring instrument such as that shown in Fig. 19 can be designed which will record changes in  $h$  of the order of the wavelength of light.

Interferometers of varying complexity, useful for photoelastic work are due to Mesnager, Vose, Fabry and Favre, the second of which is probably the most useful. Use of all types of lateral extensometer is, however, arduous if more than a few isolated spots are required and other methods of finding  $p+q$  must be investigated.

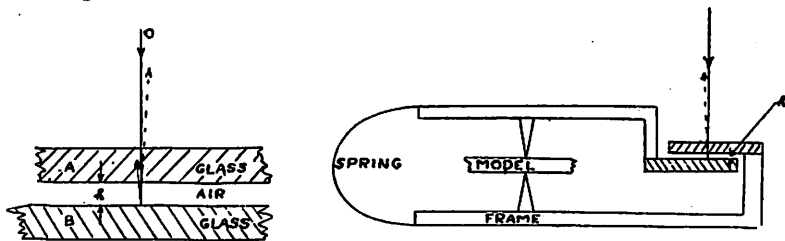


Fig. 19.

**Integration Methods.**

1. From the mathematics of elasticity, it can be shown (from the Lamé-Maxwell equations) that at any point,

$$\frac{\partial \phi}{\partial s} = \frac{q - \phi}{\rho_1}, \text{ where } \rho_1 \text{ is the radius of curvature of the } q \text{ stress trajectory}$$

and  $s$  is the distance along the  $\phi$  stress trajectory.

In Fig. 20, AOB is a free edge which the stress trajectory of the  $\phi$  set cuts at O. The lines which cut it at right angles, at C, D, . . . G, are trajectories of the  $q$  set, of which, of course, the free edge, AB, will be one. At O,  $\phi$  is zero, and the stress difference will be known at any point along OG, from the isochromatics.

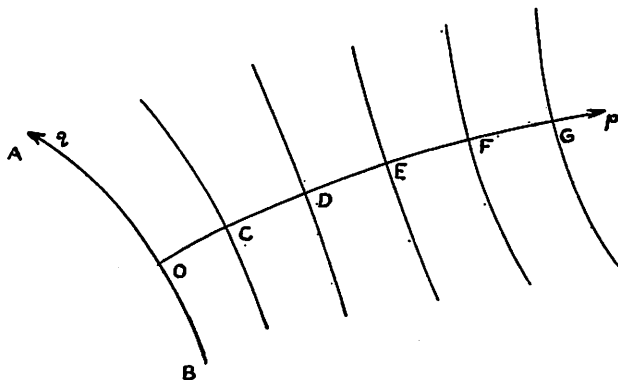


Fig. 20.

From the above formula, approximately

$$\delta\phi = \frac{q - \phi}{\rho_1} (\delta s).$$

Starting from O, the increase in  $\phi$  in moving from O to C is given by

$$\delta\phi_1 = \frac{(q - \phi)_0}{\rho_0} OC, \text{ where } \rho_0 \text{ is the radius of curvature of AB at O.}$$

Similarly from C to D,

$$\delta\phi_2 = \frac{(q - \phi)_c}{\rho_c} CD.$$

Thus, since  $\phi$  is zero at O, by adding the increments  $\delta\phi$  in moving along OG,  $\phi$  may be found at any point along the stress trajectory. The method is not very accurate, since the radii of curvature have to be found and the errors in finding  $\delta\phi$  will be additive. A way of obviating the direct use of the radii of curvature was suggested by Filon.

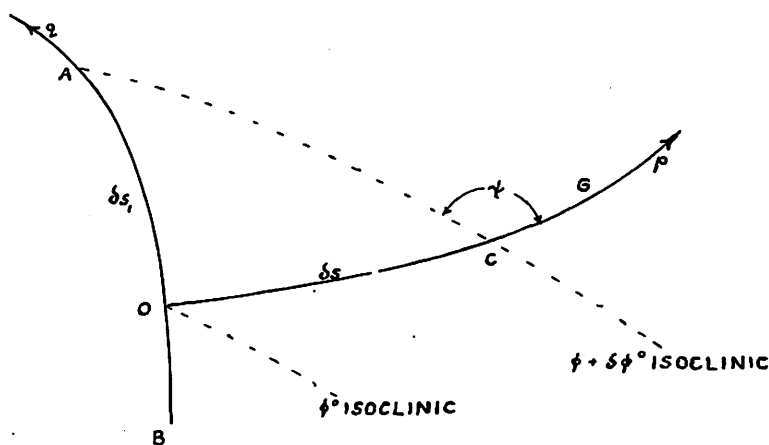


Fig. 21.

2. Consider now the isoclinics  $\phi$  and  $\phi + \delta\phi$  which pass through O and C. (Fig. 21).

$$\text{Since } e_1 = \frac{\partial s_1}{\partial \phi} = \frac{\delta s_1}{\delta \phi} \text{ approx.}$$

$$\frac{\partial \phi}{\partial s} = \frac{q - \phi}{e_1} = (q - \phi) \frac{\delta \phi}{\delta s_1}$$

$$\therefore \delta \phi = (q - \phi) \frac{\delta s}{\delta s_1} \delta \phi = -(q - \phi) \cot \psi \cdot \delta \phi$$

$$\begin{aligned} \text{since } \cot \psi &= - \frac{\delta s}{\delta s_1} \\ &= - (q - \phi) \cot \psi \delta \phi. \end{aligned}$$

Starting from O once more, therefore, and drawing in the isoclinics such that  $\delta\phi$  is small, the increments  $\delta\phi$  may be successively determined. This method is more accurate than the first one but once more has its limitations. First, the isoclinics must be plotted with great accuracy and secondly, if  $\psi$  is small, small discrepancies in its measurement will give rise to large errors in  $\cot \psi$ .

3. There is another integration method for finding the separate stresses along any straight line. If, in Fig. 22, the straight line normal to a free edge is regarded as the axis of X, it can be shown that

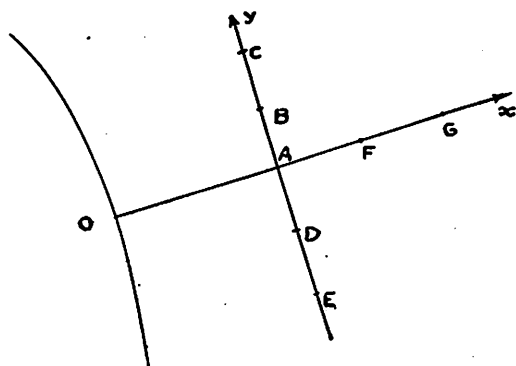


Fig. 22.

$$\frac{\partial X}{\partial x} = - \frac{\partial S}{\partial y} \text{ where } X \text{ is the normal stress in the } x \text{ direction and } S \text{ is the shear stress in the } y \text{ direction.}$$

The shear stress in the  $y$  direction is given by

$$S = \frac{1}{2} (p - q) \sin 2\alpha, \text{ where } \alpha \text{ is the angle which } p \text{ makes with } O_x.$$

Thus, from the isoclinics  $\alpha$  is found, from the fringes,  $p - q$  is found and shear stress in the  $y$  direction can be found for such points as A, B, C, D and E. By plotting a graph of these values

against distance from A, the slope at A will give  $\frac{\partial S}{\partial y}$ . Since,

approximately,  $\delta X = - \frac{\partial S}{\partial y} \delta x$ , the increments in  $X$  may be

added to give the value of  $X$  at any point along  $O_x$ . Again, from a knowledge of the nature of principal stresses,

$$p + q = 2X - (p - q) \sin 2\alpha.$$

### Relaxation Method.

This ingenious and useful method of determining  $p + q$  is based upon Laplace's Equation,  $\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) (p + q) = 0$ , which

leads to the result that the value of  $p + q$  at the centre of a small square in a stressed lamina is the mean of the values at the four corners. It involves a knowledge of  $p + q$  all round a closed circuit

in a plate, which is usually possible since along free edges  $p - q = p = p + q$ . Across other sections of the plate it may be known that a uniform tension or pure bending exists but at points where concentrated loads occur this method is not applicable.

Consider first a rectangle ADWT around which the sum of the principal stresses is known, divided into smaller squares as shown in Fig. 23. Trial values are inserted at K and L, the values of A, C, L and J meaned to give F and B, D, M and K meaned to give G. Similarly, values at O and R are deduced. The values at F, H, S and O now give a corrected value at L and those at E, G, R and N give a corrected value at K, and a cycle is complete. Using the corrected values at K and L the process is repeated until the corrections to be applied are negligible. The grid is now further subdivided and a similar process applied, resulting in improved values at F, G, etc., and values at X, Y, Z, etc. Thus the whole grid may be mapped in values of  $p + q$  determined to the accuracy of the known points. Symmetry will, of course, considerably ease computing.

Models will not, in general, be conveniently rectangular and a slight modification to the meaning of the corners of a square may be necessary.

If length  $FJ = FG = a$ , the lengths  $FA$  and  $FE$  may be expressed as  $la$  and  $ma$  respectively, where  $l$  and  $m$  are factors. The value of  $p + q$  at F is then given by

$$(p+q)_F = \frac{lm}{l+m} \left\{ \frac{(p+q)_A}{1+l} + \frac{(p+q)_C}{1+m} + \frac{(p+q)_E}{(1+l)l} + \frac{(p+q)_G}{(1+m)m} \right\}.$$

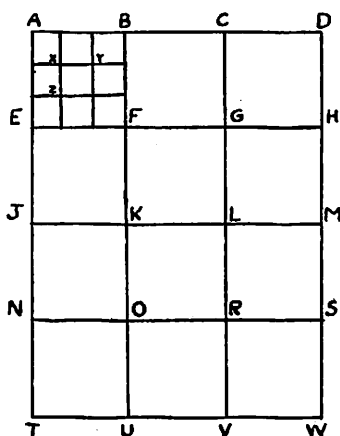


Fig. 23.

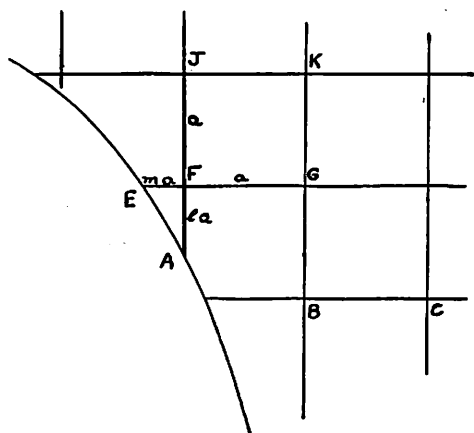


Fig. 24.

Where the stress distribution over the whole plate is required, this method is exceedingly powerful and accurate because it depends neither on an accurate plot of isoclinics, nor on the integration of many values of dubious worth.

## VI. MATERIALS.

A general description of the characteristics of various types of material follows, with a discussion of the factors which determine their value.

### Material Fringe Value.

$$\text{Retardation, } r = C (p - q) d.$$

For a given thickness,  $d$ , therefore,

$$p - q = \text{Constant} \times \text{retardation} = f \times \text{fringe order}.$$

The units used by engineers for the material fringe value,  $f$ , are lbs/in.<sup>2</sup> per inch thickness. For a material with  $f = 91$ , for example, which is  $\frac{1}{4}$ " thick, the third order fringe will represent a stress difference of 1092 lbs/in.<sup>2</sup>

### Creep.

Although immediately a model is loaded the strains experienced by it obey Hooke's Law, the strain increases with time as the load is left on. This time-creep causes not only a redistribution of the stress but alterations in the retardations occurring. Coker and Filon suggest that the law connecting strain,  $s$ , with time,  $t$ , is

$$s = a + bt^n \text{ where } a \text{ and } b \text{ are constant,}$$

$n$  depending on the material (for Xylonite it is  $1/3$ ) and this is found

to hold well for moderate values of time. It will be observed that the creep rate becomes very slow after about 30 minutes and that even though the model be loaded for only a few minutes, several days may be required for recovery.

### **Edge Effects.**

Physical changes occur at the edges of photoelastic materials with time, causing residual fringes to appear without any load being applied. For this reason, models made from materials which are prone to this time-edge effect should be examined soon after machining.

### **Bench Materials.**

*Polaroid* comprises the most convenient way of producing polarised light and is now used almost universally for the polarisers and analysers in polariscopes. It is physically a layer of rod-shaped iodoquinine sulphate crystals embedded in plastic, so manufactured that the axes of the crystals are all parallel. *Polaroid* is sold in convenient squares of 12" side, 0.010" or 0.030" thick.

*Quarter Wave Plates* are usually sheets of mica held between glass plates, split to such a thickness that a quarter wavelength retardation occurs between the doubly refracted waves for a given monochromatic beam of light. It is exceedingly difficult to split the mica to an exact thickness and a quarter wave plate may lead to small errors.

### **Model Materials.**

*Xylonite* is a mixture of cellulose nitrate and camphor. It is worked and stuck (with acetone) very readily and is suitable for the manufacture of complicated models and for use in conjunction with the portable polariscope. Severe faults are its tendency to time-creep and time-edge effects. It is cheap, however, and fairly widely used in a wide range of thicknesses, although it is somewhat insensitive compared with more recently developed materials.

*Perspex*, an acrylic plastic, is quite insensitive. It is often used, being readily stuck with chloroform, for model yokes through which the light has to pass without interference.

*Catalin 800* is a phenol-formaldehyde resin of a pale amber colour, obtainable in thick sheets and large blocks. The latter are for the manufacture of models for stress freezing, for which *Catalin* is very suitable. It creeps considerably and is subject to severe time-edge effects which may be much reduced by coating newly cut edges with petrosene wax.

*Columbia Resin 39*, an allyl-styrene resin, is glass clear and very sensitive. Creep and time-edge effects are small but due to its manufacture residual stresses occur through the thickness of the plates, making absolute normality of incident light essential. It is suitable for the frozen stress technique. It may be joined with C.R. 39 cement but the brittle nature of C.R. 39 renders machining difficult.

*Bakelite B T 61893*, is a glyptal resin made in the United States at great expense. It is, in general, a good material, clear, easily machinable, almost free from creep and time-edge effects, suitable for stress freezing, quite sensitive and has linear stress-strain and stress-fringe relationships up to 6,000 lbs/in.<sup>2</sup>

*Fosterite* is a styrene-alkyd resin of properties somewhat similar to Bakelite B T 61893 and of comparable expense, produced solely in the United States.

*Marco Resin S B 26 C*, copolymerised with styrene, is a new material produced at a reasonable price in this country. It is most suitable for frozen stress investigations and may be cast, with care, to any shape in the laboratory. The material is free from serious initial birefringence and edge and surface troubles; it is highly sensitive but creeps somewhat. It is readily machined and stuck.

---

### ACKNOWLEDGMENTS.

The author wishes to record his sincere thanks to Colonel H. T. Jessop, M.Sc., F.Inst.P., for permission to quote from his published works, and to Mr. D. L. Henry, B.A., R.C.N.C., and Mr. N. E. Gundry, R.C.N.C., for their many helpful criticisms and suggestions.

## PROPERTIES OF COMMON PHOTOELASTIC MATERIALS.

| <i>Material</i>           | <i>Fringe*<br/>Value, f.<br/>Lbs/in.<sup>2</sup><br/>in/fringe</i> | <i>Young's<br/>Modulus<br/>Lbs/in.<sup>2</sup></i> | <i>Stress-Strain<br/>Prop. Limit<br/>Lbs/in.<sup>2</sup></i> | <i>Softening<br/>Temp.<br/>Deg. C.</i> | <i>Creep</i> |
|---------------------------|--------------------------------------------------------------------|----------------------------------------------------|--------------------------------------------------------------|----------------------------------------|--------------|
| Glass                     | 850                                                                | 9,500,000                                          | 8,500                                                        | —                                      | Neglig.      |
| Xylonite                  | 285                                                                | 300,000                                            | 6,000                                                        | —                                      | Mod.         |
| Perspex                   | 600                                                                | 450,000                                            | 7,000                                                        | —                                      | Neglig.      |
| Catalin<br>800            | 48<br>1.6†                                                         | 190,000                                            | 2,500                                                        | 85                                     | High         |
| Columbia<br>Resin 39      | 91<br>35†                                                          | 290,000                                            | 7,000                                                        | 85                                     | Slight       |
| Bakelite<br>B.T. 61893    | 84<br>3.2†                                                         | 630,000                                            | 5,500                                                        | 110                                    | Slight       |
| Fumaric<br>Fosterite      | 4.5†                                                               | †2,900                                             | —                                                            | 100                                    | Neglig.      |
| Marco Resin<br>S.B. 26 C. | 8.1†                                                               | †4,700                                             | —                                                            | 96                                     | Mod.         |

\* At room temperature.

† At the softening temperature.

## VII. BIBLIOGRAPHY.

1. COKER, E. G. AND FILON, L. N. G. A Treatise on Photoelasticity. Cambridge University Press. 1930.
2. FROCHT, M. M. Photoelasticity. Vols. I and II. John Wiley & Sons. 1941 and 1948.
3. JESSOP, H. T. AND HARRIS, F. C. Photoelasticity. Cleaver-Hume. 1949.
4. FILON, L. N. G. Photoelasticity for Engineers. Cambridge University Press. 1936.
5. HENDRY, A. W. An Introduction to Photoelastic Analysis. Blackie & Sons. 1948.
6. JESSOP, H. T. The Scattered Light Method of Exploration of Stresses in Two- and Three-Dimensional Models. The British Journal of Applied Physics, Vol. 2, pp. 249-260. September, 1951.
7. SUGARMAN, B., MOXLEY, G. O., AND MARSHALL, I. A. A Castable Polyester Resin for Photoelastic Work. British Journal of Applied Physics, Vol. 3. July, 1952.
8. PAFFETT, J. A. H. The Application of Photoelastic Methods to Ship Design Problems. Proc. North-East Coast Inst. of Engineers and Shipbuilders. October, 1949.
9. TARDY, H. L. Méthode Pratique d'Examen et de Mesure de la Birefringence des Verres d'Optiques. Rev. Opt. (théor. instrum.) 8, pp. 235, 1929.

## A.E.S.D. Printed Pamphlets and Other Publications in Stock.

---

An up-to-date list of A.E.S.D. pamphlets in stock is obtainable on application to the Editor, *The Draughtsman*, Onslow Hall, Little Green, Richmond, Surrey.

A similar list is also published in *The Draughtsman* twice a year.

Readers are asked to consult this list before ordering pamphlets published in previous sessions.

## List of A.E.S.D. Data Sheets.

1. Safe Load on Machine-Cut Spur Gears.
2. Deflection of Shafts and Beams.
3. Deflection of Shafts and Beams (Instruction Sheet). } Connected.
4. Steam Radiation Heating Chart.
5. Horse-Power of Leather Belts, etc.
6. Automobile Brakes (Axle Brakes).
7. Automobile Brakes (Transmission Brakes). } Connected.
8. Capacities of Bucket Elevators.
9. Valley Angle Chart for Hoppers and Chutes.
10. Shafts up to 5½-in. diameter, subjected to Twisting and Combined Bending and Twisting.
11. Shafts, 5½ to 26 inch diameter, subjected to Twisting and Combined Bending and Twisting.
12. Ship Derrick Booms.
13. Spiral Springs (Diameter of Round or Square Wire).
14. Spiral Springs (Compression).
15. Automobile Clutches (Cone Clutches).
16. " " (Plate Clutches).
17. Coil Friction for Belts, etc.
18. Internal Expanding Brakes. Self-Balancing Brake Shoes (Force Diagram).
19. Internal Expanding Brakes. Angular Proportions for Self-Balancing. } Connected.
20. Referred Mean Pressure Cut-Off, etc.
21. Particulars for Balata Belt Drives.
22. 7/8" Square Duralumin Tubes as Struts.
23. 1" " " "
24. 1½" Sq. Steel Tubes as Struts (30 ton yield).
25. 1½" " " " (30 ton yield).
26. 1" " " " (30 ton yield).
27. 1½" " " " (40 ton yield).
28. 1½" " " " (40 ton yield).
29. 1" " " " (40 ton yield).
30. Moments of Inertia of Built-up Sections (Tables).
31. Moments of Inertia of Built-up Sections (Instructions and Examples). } Connected.
32. Reinforced Concrete Slabs (Line Chart).
33. Reinforced Concrete Slabs (Instructions and Examples). } Connected.
34. Capacity and Speed Chart for Troughed Band Conveyors.
35. Screw Propeller Design (Sheet 1, Diameter Chart).
36. " " " (Sheet 2, Pitch Chart).
37. " " " (Sheet 3, Notes and Examples). } Connected.
38. Open Coil "Conical" Springs.
39. Close Coil Conical Springs.
40. Trajectory Described by Belt Conveyors (Revised 1949),
41. Metric Equivalents.
42. Useful Conversion Factors.
43. Torsion of Non-Circular Shafts.
44. Railway Vehicles on Curves.
46. Coned Plate Development.
47. Solution of Triangles (Sheet 1, Right Angles).
48. Solution of Triangles (Sheet 2, Oblique Angles).
49. Relation between Length, Linear Movement and Angular Movement of Lever (Diagram and Notes).
50. " " " " " " (Chart).
51. Helix Angle and Efficiency of Screws and Worms.
52. Approximate Radius of Gyration of Various Sections.

53. Helical Spring Graphs (Round Wire) } Connected.
54. " " " (Round Wire) }
55. " " " (Square Wire) }
56. Relative Value of Welds to Rivets.
58. Graphs for Strength of Rectangular Flat Plates of Uniform Thickness.
59. Graphs for Deflection of Rectangular Flat Plates of Uniform Thickness.
60. Moment of Resistance of Reinforced Concrete Beams.
61. Deflection of Leaf Spring.
62. Strength of Leaf Spring.
63. Chart showing Relationship of Various Hardness Tests.
64. Shaft Horse Power and Proportions of Worm Gears.
65. Ring with Uniform Internal Load (Tangential Strain) } Connected.
66. Ring with Uniform Internal Load (Tangential Stress) }
67. Hub Pressed on to Steel Shaft. (Maximum Tangential Stress at Bore of Hub).
68. Hub Pressed on to Steel Shaft. (Radial Gripping Pressure between Hub and Shaft).
69. Rotating Disc (Steel) Tangential Strain } Connected.
70. " " " Stress }
71. Ring with Uniform External Load, Tangential Strain } Connected.
72. " " " Stress }
73. Viscosity Temperature Chart for Converting Commercial to Absolute Viscosities. } Connected.
74. Journal Friction on Bearings.
75. Ring Oil Bearings.
76. Shearing and Bearing Values for High Tensile Structural Steel Shop Rivets, in accordance with B.S.S. No. 548/1934.
78. Velocity of Flow in Pipes for a Given Delivery } Connected.
79. Delivery of Water in Pipes for a Given Head }
80. (See No. 105).
81. Involute Toothed Gearing Chart.
83. Variation of Suction Lift and Temperature for Centrifugal Pumps.
89. Curve Relating Natural Frequency and Deflection
90. Vibration Transmissibility Curve for Elastic Suspension } Connected.
91. Instructions and Examples in the Use of Data Sheets, Nos. 89 and 90. }
92. Pressure on Sides of Bunker.
- 93-4-5-6-7. Rolled Steel Sections.
- 98-99-100. Boiler Safety Valves.
102. Pressure Required for Blanking and Piercing.
103. Punch and Die Clearances for Blanking and Piercing.
104. Nomograph for Valley Angles of Hoppers and Chutes.
105. Permissible Working Stresses in Mild Steel Struts with B.S. 449, 1948.
106. Compound Cylinder (Similar Material) Radial Pressure of Common Diameter (D1).

(Data Sheets are 3d to Members, 6d to others, post free.)

Orders for Pamphlets and Data Sheets to be sent to the Editor, *The Draughtsman*, cheques and orders being crossed "A.E.S.D."